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## ELECTROSTATIC FORCE AND COULOMB'S LAW

## INTRODUCTION

The fundamental relation expressing the force F acting between two electric charges, $q_{1}$ and $q_{2}$, and separated by a distance $r$ is given by Coulomb's law:

$$
F=\frac{k q_{1} q_{1}}{r^{2}}
$$

It is interesting to notice that this expression is formally similar to Newton's law governing the gravitational attraction between two massive bodies, although there are some fundamental differences. To start with, it has been established that the electrostatic force is orders of magnitude stronger than the gravitational force. In spite of this, the effects of the electrostatic force are not easily observed at the macroscopic level, in contrast with the easily observed effect of the gravitational pull. The main reason for this is that electric charges come in two types (so called "positive" and "negative"). When felt by a third object, the forces due to two equal but opposite charges in close proximity (like a Hydrogen atom) cancel each other. Gravitational masses, however, come with one sign only (we have never observed an object carrying a "negative" mass), gravitational pulls of individual elementary particles forming a large body add up, and no object intervenes to cancel the gravitational force of a given body. Even so, the electrostatic force plays a most fundamental role in shaping the world as we know it, is responsible for holding the negative atomic electrons around the positive nucleus, and it governs the whole spectrum of chemical and, consequently, biological processes.

In this experiment, you will verify the validity of Coulomb's law by measuring the force acting between two charged spheres. The force will be measured by means of a torsion balance: in your setup you will produce a repulsion between a fixed sphere and another one mounted on a rod, counter-balanced and suspended from a thin torsion wire. To perform the experiment both spheres are charged and kept at a given distance. The electrostatic repulsion between the spheres causes the torsion wire to twist, the twist angle being proportional to the force acting upon the sphere, $F \sim \theta$.

To deposit a certain amount of charge onto the spheres, you will use a power supply, an instrument capable of providing a stable voltage at one of its terminals. By touching the spheres with the power supply output probe, the sphere will be brought to the same voltage as the supply output, and a certain amount of charge will be deposited onto the sphere. You might already have learned, or will learn very soon, that the amount of charge $q$ required to bring a body to a certain $V$ is proportional to the voltage itself, i.e.

$$
q=C V
$$

where the constant of proportionality $C$ is called capacitance, which is a characteristic property
of any given body. One can obtain an intuitive appreciation of the meaning of capacitance by thinking in hydrostatic terms: if you have two containers, a slim one and a fat one, filled to an equal height, the liquid at the top has the same (gravitational) potential energy for either container. At the same time, more liquid is required to fill the fat container to the desired height: the fat container has more capacity.... If you replace the quantity of liquid with amount of charge and the gravitational potential with electrical potential, the comparison stands.

Experiments with the Coulomb balance could, in principle, be quite accurate; yet, as with any quantitative electrostatic experiment, spurious effects can intervene to spoil the measurement. A charged shirtsleeve, an open window, an excessively humid day - any of these effects and many more can affect your experiment. For your setup the following precautions should be taken:

- The experiment should be performed during part of the year when the humidity is low unfortunately that is not the case with our summer workshop course. The winter is an ideal time to do this experiment. We will have the air conditioners and dehumidifiers on prior to your starting this experiment. However, we must turn them off when you start the experiment to avoid air currents, which have been shown in the past to disturb the performance of the torsion balance.
- The experiment should be performed in a draft free room, and the equipment installed on an insulating table and away from the walls. (Were any conducting surface near your equipment, image charges would arise in it, with the effect of distorting your measurements.)

In addition, you should take the following precautions:

- When performing the measurements, stand at the maximum comfortable distance away from the balance. This will minimize the effects of static charges that may collect in your clothing. The effect is particularly severe if you are wearing synthetic fabrics.
- When charging the spheres, turn the power supply on and set it at the desired voltage, charge the spheres, then immediately turn the supply down and off. The high voltage at the terminals of the supply can cause leakage currents, which will affect the torsion balance.
- When charging the spheres, hold the charging probe near the end of the handle (i.e. far from the metal tip), so that your hand is as far from the spheres as possible.
- Avoid touching the rods supporting the spheres: fingerprints produce a conductive coating, which causes charge leakage.
- To minimize the effect of slow unavoidable charge leakage into the air, perform measurements as quickly as possible after charging. Recharge the spheres before each measurement.


## APPARATUS

Pasco Coulomb Balance apparatus, Pasco 6 kV power supply with sphere charging probe.

## WHAT TO DO

The torsion balance assembly is shown schematically in Fig. 1, which gives a side and a top view. Before starting your measurements you should verify the following conditions: (ask your Instructor if you have any questions)

- the counterweight vane balancing the suspended sphere should be fairly horizontal, and the mark at its center should be aligned with the corresponding mark on the index arm.
- the mark on the torsion knob at the top of the balance should be aligned with the zero degree mark.
- when bringing the second sphere (the one mounted onto the sliding guide) as close as possible to the suspended sphere (see Fig. 2), the spheres should be aligned, both vertically and laterally.
- when the two spheres are barely touching, the pointer on the sliding arm should read 3.8 cm on the centimeter scale (given that 3.8 cm represents the diameter of the spheres, this will ensure that the reading on the centimeter scale will reflect the distance between the center of the spheres).


Fig. 1. Torsion balance assembly. a) Side view. b) Top see-through view (the torsion knob assembly is shown with dotted lines).

If any of the above conditions is not met, ask your Instructor for assistance.

CAUTION:
THE WHOLE TORSION BALANCE ASSEMBLY IS EXTREMELY DELICATE. YOU SHOULD AVOID HANDLING IT AS MUCH AS POSSIBLE. IN PARTICULAR, YOU SHOULD BE CAREFUL NOT TO HIT THE HARDLY VISIBLE WIRE TO WHICH THE BALANCED SPHERE IS SUSPENDED.


You are now ready to start your measurements.

Fig. 2. Complete Coulomb balance assembly including torsion balance and independent sphere on sliding track.

## 1. Force versus distance

a. In order to start from a known situation, fully discharge the two spheres by touching them with a grounded probe. Then move the sliding sphere as far as possible from the suspended sphere. With the spheres at maximum separation, turn on the power supply to 6 kilovolts $(\mathrm{kV})$ and charge both spheres by touching them with the power supply probe. Turn off the power supply immediately after charging the sphere.
b. Position the sliding sphere at 20 cm . Notice that the suspended sphere will tend to move away from the other sphere, under the effect of the electrostatic repulsion. This will cause the whole suspended assembly to rotate away from its equilibrium position. The rotation of the assembly is proportional to the force acting upon the sphere. When equilibrium is reached the torsion of the wire counterbalances the repulsive force. Adjust (slowly) the torsion knob on top of the balance as necessary to bring the pendulum back to the zero position. Record the angle. HINT: IN A HUMID ROOM, CHARGE WILL BE LEAKING OFF THE SPHERES CONTINUOSLY, SO TIME IS OF THE ESSENCE! DO NOT DALLY! YOU MAY HAVE TO START OVER.
c. You should then repeat the above procedure to convince yourself that you can achieve reproducible results: discharge the spheres, zero the torsion balance, then separate the spheres to their maximum distance, recharge them to the same 6 kV voltage, and reposition them to the 20 cm distance. Measure the torsion angle and record it. Repeat the procedure a few times, and verify that the torsion angle is reproduced to $10 \%$ or better.
d. Repeat the procedure for at least four other values of the separation between the spheres, e.g. for $16,13,10,8 \mathrm{~cm}$. For each separation measure the angle two or three times, and calculate the mean value of the angle for each setting.
e. You can now verify the $1 / r^{2}$ dependence of the force. Use Excel or Data Studio to plot your angles versus the inverse square of the distance: you should notice a nearly linear behavior, with the deviations from linearity being large for the smaller values of the distance (= larger values of $1 / r^{2}$ ). The reason for the deviation is that, at short distances, the charged spheres cannot be thought anymore as point charges. A charged conductive sphere, if it is isolated from other electrostatic influences, will act as a point charge. The charges distribute themselves evenly on the surface of the sphere, so that the center of the charge distribution is just at the center of the sphere. However, when two charged spheres are separated by a distance that is not large compared to the size of the spheres, the charges will redistribute themselves on the spheres so as to minimize the electrostatic energy. The force between the spheres will therefore be less than it would be if the charged spheres were actual point charges.
f. A correction factor $B$, can be used to correct for this deviation. Simply multiply each value of $\theta$ by $1 / B$, where

$$
B=1-4 \frac{a^{3}}{R^{3}},
$$

where $a$ equals the radius of the spheres and $R$ is the separation between the center of the spheres. To correct your data (which can be easily accomplished with EXCEL):

1. Calculate the correction factor $B$ for each of the separation $R$ values that you used.
2. Multiply each of your collected values of $\theta$ by $1 / B$.
3. Reconstruct your graphs relating force and separation, but this time use the corrected value of $\theta$. Make your new plot is on the same graph as your original plot. Distinguish the plots somehow. How does the correction factor affect your results?

## 2. Force versus charge

In the second part of your measurements, you will verify the dependence of the force upon the charge, $F \sim q_{1} q_{2}$. As you do not actually perform a measurement of the charge deposited upon the spheres, you should remember that the charge is proportional to the voltage (Eq. 2), therefore you can express your results in terms of voltage applied to the spheres vs. the resulting force. In order to perform the experiment, you can follow two different procedures, the choice among the two being left to your initiative.
a. The first procedure consists in charging the suspended sphere at a constant voltage (e.g. 6 kV ), and perform a series of force measurements for different values of the second sphere voltage (e.g. 1, 2, 3, 4, 5, 6 kV ). Obviously the measurements should be performed with the two spheres always at the same distance (e.g. 10 cm ), although you should remember that, when charging the spheres, they should be kept at their maximum separation. The linearity of the force vs. charge can be verified by plotting the deflection angle vs. the voltage of the sliding sphere. A possible drawback of this procedure is that the suspended sphere, originally charged at 6 kV , can shed some of its charge throughout the execution of the
measurement (can you think of a procedure that would avoid such a potential cause of error?)
b. In the alternate procedure, you perform your measurements with both spheres charged at the same potential ( $1,2,3, \ldots \mathrm{kV}$ ). This way you have a better control upon the sphere charge, but you should be careful how to plot your data. (Remember, the force is proportional to the product of the charges....)

Plot your data indicating how the force changes with the product of the two charges. Discuss how well the data agree with a linear line. Discuss possible sources of error.

## 3. Charge distribution among identical bodies

Suppose you have two identical conductors, like the two charged spheres, the first carrying a certain charge $q$, the second being free of charge. If you bring the two bodies into contact, a certain amount of charge will migrate from the charged body to the other one. If the two bodies are identical, in the final configuration the charge will be equally distributed among the two bodies, i.e. each body will carry a charge $q / 2$. (You could easily prove this by remembering that $q=C V$, two identical bodies will have the same value of capacitance $C$, and, when brought into contact, the two bodies will reach the same potential. The hydrostatic model can be of help again: if you have two identical containers, one filled to a given level, the other empty, when brought into communication they will both end up filled at half the original level).

In the final set of measurements, you should devise a procedure, which will prove the equidistribution of charge discussed above. In practice, what you need to do is to bring the two spheres to the same potential, measure the force at a given distance, discharge one of the spheres, recharge it by bringing it into contact with the second sphere and re-measure the force at the same distance. Keep doing this at least three times. What happens? Write down your procedure very carefully so another student could follow it. What do you predict will occur? How well does you data agree with your predictions.

## PLEASE CLEAN UP YOUR LAB AREA!

## TURN OFF THE POWER SUPPLY!

